Elutriation of Solid Particles From a Dense-Phase Fluidized Bed

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The elutriation of fines from fluidized beds of two- and multiparticle systems is investigated. Materials used are glass beads and coal powder with air and helium as the elutriating gases.

A specific elutriation rate constant which takes into consideration the particle stratification and thus is nearly independent of the bed dimensions is employed in the correlation of the elutriation of fines.

The proposed generalized correlation applicable to a system of two particle sizes and to more complex ones is presented. Recorrelation of pertinent literature data by the dimensionless equation indicates good agreement in spite of a diversity of bed geometries. The effect of fluidized-bed parameters on the rate of elutriation is examined, and applications and limitations of the correlation are shown.

Elutriation is a process whereby smaller particles are continuously removed by means of a fluid current from a bed composed of a spectrum of particle sizes. The understanding of the process of elutriation of solid particles has in recent years become progressively important not only owing to the increased use of fluidized-bed technique in industrial plant operations but also because of the convenience of this process as a method of particle separation. In a continuous fluidized-bed operation the fine particles may be originally present, or they may in time be produced from large-size particles as a result of mechanical attrition or breakdown due to internal thermal stresses. The carry-over of fines must be either replaced with fresh particles or returned to the system through separation equipment such as cyclones or electrostatic precipitators. If the entrained solid particles are allowed to escape into the transfer line, a reduction in flow capacities and an erosion of pipelines will result. Furthermore the constantly changing particle-size distribution in the fluidized bed may alter the rate and extent of mass and heat transfer. These facts necessitate a thorough investigation of the rate of elutriation upon which design calculation can be made. Finally the importance of elutriation may be clearly seen in the possibility of adapting the fluidized-bed technique to nuclear reactors, where contamination by radioactive fines is a serious problem.

Several investigators (*2, 5, 7*) who have worked on two-particle size systems have made important contributions to the understanding of the mechanism of elutriation. The experimental results of previous investigators agree qualitatively in major parts and differ in minor aspects. Leva (*5*) and Osberg and Charlesworth (*7*) studied the rate of elutriation from beds of two different particle sizes. They could express their data by a first-order rate equation, $C = C_o e^{-k\theta}$. The system variables, such as gas velocity, fine particle size and concentration, solid and gas properties, etc., were empirically related to the elutriation constant. Leva considered the gas velocity as the driving force in elutriation and found the rate constant to be proportional to $u_o^{4.0}$. Osberg and Charlesworth, on the other hand, correlated k in terms of particle free-fall terminal velocity, the difference between the value of the linear gas velocity and particle free-fall terminal velocity being the elutriation driving force. Leva found that the fines concentrations ranging from 20 to 5% had a negligible effect on the elutriation constant; however Osberg and Charlesworth indicated that below 5% the fines concentration had a significant effect. The effect of coarse-particle size on the elutriation rate constant was reported to be negligible by Leva, whereas Osberg and Charlesworth noted a significant effect when the fines concentration was less than 1%. A discussion of bed mobility for multiparticle size system is given by Matheson (*6*) and Trawinski (*11*). Andrews (*1*) considered the kinetics of the system and a statistical function of the freeboard. The information currently available from these studies is only qualitative and rather fragmentary and therefore cannot support a generalization for prediction of elutriation rate.

The object of this investigation is to augment the present understanding of the mechanism to include multiparticle size systems and to propose a generalized correlation for the elutriation rate constant.

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EXPERIMENTAL METHOD

Equipment

Figure 1 is a schematic diagram of the equipment. A 4-in. I.D. glass column 72 in. high and a 2-in. standard steel pipe 82 in. high were used as the fluidization equipment. A cyclone separator for recovery of fines was attached to the top of the column. Glass containers were placed below this cyclone to collect the fines carried out from the column. A piece of filter cloth placed between the column and the base served as the gas distributor. A pressure tap at the base of the column connected to a manometer was used to indicate the gas pressure as it entered the column. The air used for elutriation of the fines was metered through a rotameter before entering the column.

Materials

Scotchlite glass beads and bituminous coal powder were the solids; air and helium were the gases used. The sizes of glass beads used were 0.0028, 0.0039, 0.0058, and 0.0110 in. in diameter. The material had an absolute density of 156 lb./cu. ft. The coal powder had average particle sizes of 0.0281, 0.0141, 0.0053 and 0.0038 in. in diameter. The absolute density of the coal powder was 81 lb./cu. ft.

Procedure

The gas was introduced at a velocity well below the free-fall velocity of the smallest particle in the bed and was kept at this rate for several minutes to assure mixing of the various particle sizes of the bed. The gas rate was then adjusted until the desired velocity was obtained. The carry-over of fines was collected by the cyclone for a suitable time interval and then separated into the various particle sizes by means of screens.

Occasional examination of the solids under the microscope indicated that no appreciable attrition had occurred. The electrostatic charges which accumulate on the particles during the course of elutriation frequently hinder considerably the accuracy and reproducibility of the results. To minimize this effect, a ground wire was inserted into the fluidized bed in addition to the wiring of the entire column from outside for grounding. This was found satisfactory in obtaining the reproducible data.

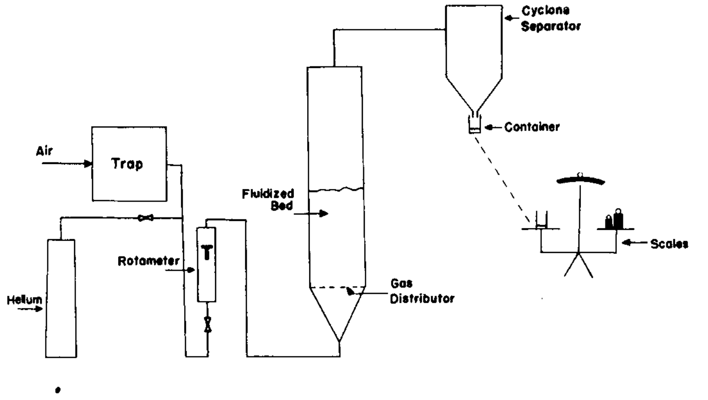

Fig. 1. Schematic diagram of equipment.

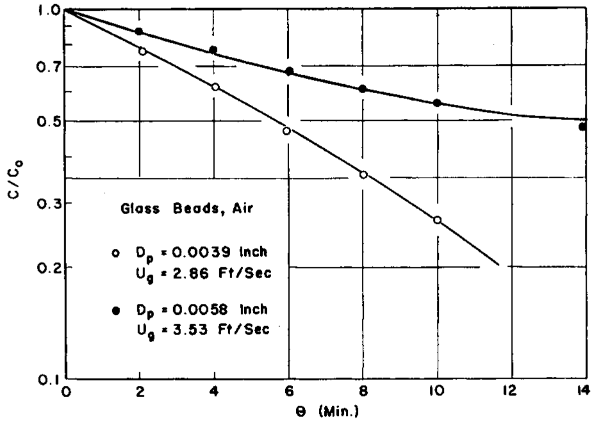

Fig. 3. Fines concentration vs. time.

MECHANISM

In a fluidized bed of comparatively small diameter the solid particles move predominantly upward in the center of the column and considerably less vigorously downward near the vessel wall. In a large-diameter column, however, the pattern of flow is somewhat less well coordinated, and local vortices may prevail. At any rate the resulting fluidized bed is characterized by a turbulent bubbling surface similar to that of a boiling liquid. The elutriation of the fines from a bed composed of a size spectrum will occur only when the gas velocity above the bed exceeds the free-fall velocity of the fine particles. Since the actual local gas velocity in the bed is considerably higher than the superficial gas velocity, the gas bubbles carry with them the fines as well as coarse particles through the fluidizing bed. The kinetic energy of these particles moving through the bed can be ascribed to friction force and buoyance force of the gas current and forces due to impact from the particles below. As the particles reach the surface of the bed, a sudden eruption of gas bubbles takes place, throwing particles up above the bed surface boundary. This sudden expansion of the gas reduces the gas velocity to below the free-fall terminal velocity of the coarse particles, causing the coarse particles to fall back into the bed. However the gas velocity is still larger than the free-fall velocity of the fines, and these will be carried out of the column. Thus above the bed Stokes's-law forces will prevail except in some instances when occasional collisions occur between the particles. In addition to the particle collisions, the development of a pronounced velocity gradient of the gas above the bed will also act to bring some fines situated near the column wall back into the bed. This effect can be seen from the velocity gradient of the gas above the bed as shown in Figure 2. The gas-velocity profiles immediately above the bed (A) and significantly farther above the bed surface (B) are indicated in the figure. For a spherical particle traveling above the bed without collision, the particle velocity can be expressed as

$$u = (u_g - u_t)(1 - e^{-\alpha\theta}) + u_o e^{-\alpha\theta} \quad (1)$$

where α equals $18\mu/\rho_s D_p^2$. Since the value of $e^{-\alpha\theta}$ is very small, especially for small particles, $u \cong u_g - u_t$ and is practically independent of time. Accordingly, as the velocity gradient becomes progressively developed, the fine particles along the wall will be subjected to low-velocity gas and thus fall back into the bed while the particles in the center are being elutriated. A certain height is required for the gas to develop the velocity gradient fully. This height, called *critical height,* will be discussed later in some detail.

Since the rate of stratification of fine particles is considerably greater than that of coarse particles, accumulation of the fines on the surface of the bed will occur until a concentration gradient is reached throughout the bed with a maximum accumulation of the fines at the surface. Therefore during the elutriation the rate of fines thrown up above the bed as a result of eruption of gas bubbles will depend greatly on the population of the fines on the bed surface. Leva (5) employed a first-order rate equation to describe the rate of elutriation. If C is the concentration of the fines in the bed at any time, then

$$\frac{dC}{d\theta} = -kC \quad (2)$$

where k is a proportionality constant and is designated the elutriation constant. As the elutriation progresses, a reduction in fines population takes place which is in turn partially made up by the fines stratified from below. A plot of C/C_o vs. θ, as shown in Figure 3, indicates that this relationship tends to deviate from linearity at large values of θ. This is due to the fact that the bed height is continually decreasing, and therefore the time of stratification required for the fine particles through the bed interior to the surface will be decreasing. At the same time the fines concentration is also decreasing, which may increase elutriation time. The curve in Figure 3 therefore can be either concave upward or downward, depending on the operating conditions. Thus when the amount of elutriation becomes appreciable, the line shows a substantial curvature. Since the effective time of stratification of the fines particles is governed by the height of the bed on one hand and the simultaneous decrease of fines concentration on the other, the effect of the bed height on the value of k cannot be accurately evaluated from Figure 3 unless the fines concentration is maintained constant throughout the operation. Consequently the change in fines concentration in the bed should be considered to be proportional to fines concentration and inversely proportional to the bed height through which the

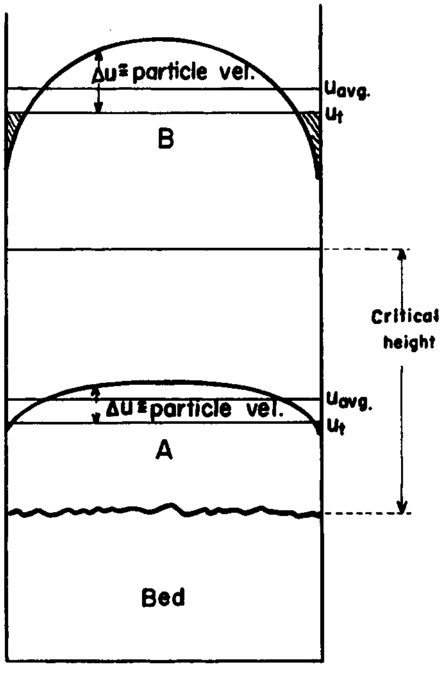

Fig. 2. Velocity distribution of gas above bed.

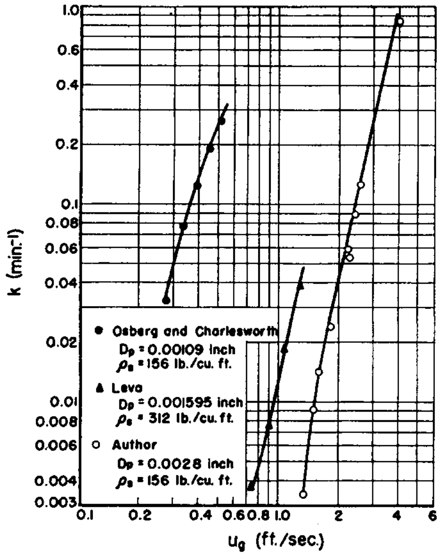

Fig. 4. Effect of linear gas velocity on elutriation constant.

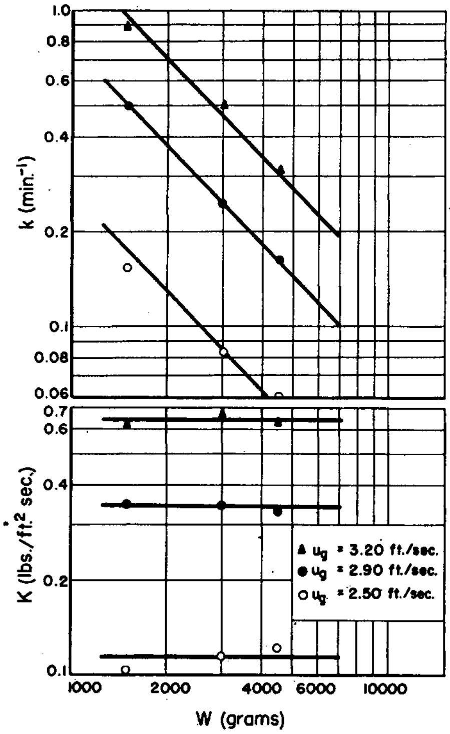

Fig. 5a. Effect of bed weight on elutriation constant and specific elutriation constant.

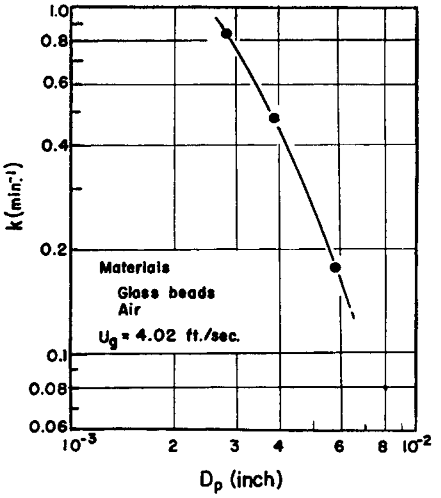

Fig. 5b. Effect of particle size on elutriation constant.

fine particles must travel in order to emerge to the surface. For convenience in measurement the bed weight will be used instead of the bed height for the expression of the rate of elutriation. If A is the bed cross-sectional area, it follows that

$$\frac{dC}{d\theta} = -K\frac{A}{W}C \qquad (3)$$

where K is a proportionality constant and has the unit of pounds per square foot per second. Yagi and Aoji (*12*) employed the same equation and attempted a correlation by means of a dimensionless empirical equation.

DIMENSIONAL ANALYSIS

Since the rate of elutriation is affected by the solid-particle characteristics, the gas properties, and the particle velocity above the bed, a dimensional analysis may be applied:

$$K = f[D_p, u_t, (u_g-u_t), \rho_g, (\rho_s-\rho_g), \mu, g] \qquad (4)$$

The resulting equation is

$$\frac{K}{\rho_g(u_g-u_t)} = \beta\left[\frac{(u_g-u_t)^2}{gD_p}\right]^a\left[\frac{D_p u_t \rho_g}{\mu}\right]^b \left[\frac{\rho_s-\rho_g}{\rho_g}\right]^c\left[\frac{u_g-u_t}{u_t}\right]^d \qquad (5)$$

The examination of the experimental data indicated that the power of $[(u_g-u_t)/u_t]$ was negligible, and the group was eliminated.

In all the free-fall terminal-velocity calculations Stokes's law or a modification of it was used. For particles with a Reynolds number less than 2 Stokes's law is applicable:

$$u_t = \frac{g D_p^2(\rho_s-\rho_g)}{18\mu} \qquad (6)$$

A correction-factor chart (*8*) was used with Stokes's law to correct for deviation as the particle size increased.

For Reynolds numbers larger than 2 a modified Stokes's equation is applicable:

$$u_t = \frac{0.153\, g^{0.71} D_p^{1.14}(\rho_s-\rho_g)^{0.71}}{\rho_g^{0.29}\mu^{0.43}} \qquad (7)$$

A correction was made for nonspherical particles according to the following equation (*9*):

$$Y = 0.843 \log\left(\frac{\phi}{0.065}\right) \qquad (8)$$

The calculated value of the free-fall terminal velocity is multiplied by Y to make the correction.

CORRELATION AND DISCUSSION

The variables affecting the elutriation-rate constant can be classified into three general groups: gas variables (gas velocity, gas density, gas viscosity), bed variables (fine particle size, solid particle density, bed height, fines concentration, coarse particle size), and column variables (column diameter, freeboard, baffling).

Gas Velocity

A significant effect of gas velocity on the value of the elutriation constant can be observed from Figure 4. The data of different investigators show that $k \propto u_g^{4.0-7.0}$. As indicated previously, the major factor affecting the rate of elutriation is the difference between the magnitude of the gas velocity and the particle free-fall terminal velocity, (u_g-u_t). When the linear gas velocity is lower than the particle free-fall terminal velocity, the gas carries only a negligible amount of fines from the column; although the impact from below, induced by collisions with other particles, may throw up some particles above the bed, they will in due course fall back into the bed. However when the gas velocity exceeds the fine-particle terminal velocity, the gas stream will remove a significant number of particles from the column. If on the other hand the gas velocity is larger than the terminal velocity of one of the fine particle sizes in a multiparticle size system but lower than the terminal velocity of a second fine size, a significant amount of the smaller size alone will be removed from the bed. If the gas velocity is increased above the terminal velocity of the second fine particle size, a considerable amount of both sizes will be removed. Below is shown a typical run

Table 1

D_p, in.	Initial weight of particles, g.	u_t, ft./sec.	Grams of fines removed in 1 min. $u_g = 1.45$ ft./sec.	$u_g = 2.21$	$u_g = 4.02$
0.0028	450	1.07	2.47	25.8	207
0.0039	450	1.88	0	4.45	145
0.0058	450	2.92	0	0	74.1
0.0110	4050	——	0	0	0

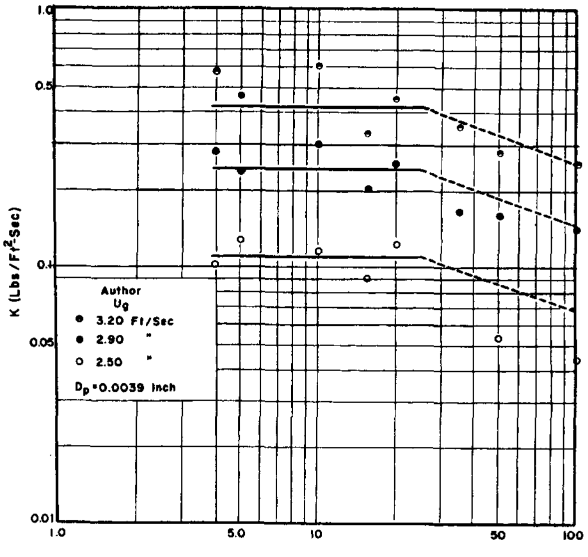

Fig. 6. Effect of fines concentration on the specific elutriation constant.

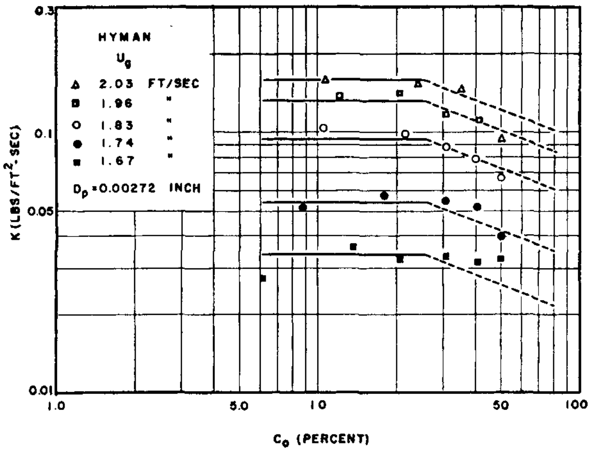

Fig. 7. Effect of fines concentration on the specific elutriation constant.

for a four-particle size system (Table 1).*

From this chart one observes that a considerable amount of fines was collected for those fines having a terminal velocity below the linear gas velocity and negligible amount for those fines having a terminal velocity above the gas velocity.

The velocity distribution in fully developed flow of the gas above the bed is shown in Figure 2. The velocity in the middle of the column of the gas reaches a maximum value of 1.25-2.0 times the average velocity. Near this region of maximum velocity the actual gas velocity might be larger than the terminal velocity of the coarse particle sizes, and therefore it is possible to remove a small amount of the coarse particles even though the average gas velocity is smaller than the terminal velocity of the particle.

Bed Weight

The elutriation constant for a given size column is shown to be inversely proportional to the bed weight in Figure 5*a*, where k is plotted against the bed weight at constant velocity. The concentration of fines is kept constant by taking the initial slope of the C/C_o vs. θ plot for different heights.

During elutriation of fine particles from a fluidized bed a concentration gradient is formed in the bed with the largest concentration of the fines found near the surface of the bed. The fine particles must travel to the surface of the bed before they can be removed, and the time of travel of the fines will be increased as the bed weight is increased. An increase in the travel time of the fines will reduce the concentration gradient in the bed, and this will decrease the value of the elutriation constant.

In the same figure the specific elutriation constant is plotted against the bed weight for comparison. A negligible effect of bed weight is seen on K. Knowing the value of K, one can derive an equation to calculate the amount of fines removed from a bed in a specified time.

Fine Particle Size and Concentration

For a given gas rate the elutriation rates were observed to increase markedly with decreasing fines diameter. The value of k thus decreases sharply as fine size is increased, as illustrated in Figure 5*b*. This is due to the fact that the terminal velocity of the fine particles increases with increasing particle size, thereby reducing the driving force. The increase in fine-particle density will also cause a similar result.

The effect of fines concentration in a two-particle size system is shown in Figures 6 and 7, where the specific elutriation constant is plotted against the fines concentration in the bed. The fines concentration has a small effect on the modified elutriation constant until a concentration of approximately 25% is used, and then the value of the modified elutriation constant seems to decrease as the fines concentration increases. The ratio of individual values of K to the average value obtained from Figures 6 and 7 was plotted in Figure 8. Although the trend is indicated, a further investigation is desirable to substantiate the variation of specific elutriation constant with respect to fines concentration.

In a multiparticle size system the effect of fines concentration can be considered to be similar to that for a two-particle size system because the correlation of the multiparticle size-system data showed that each of the fine components, when considered with coarse size as a two-particle size system, retained their individuality and acted as a two-particle size system. Therefore as long as the composition of each of the fine-particle sizes, when considered with the coarse-particle size, remains below 25%, the presence of the third particle size has no bearing on the rate of elutriation of that particular individual fine-particle size. This is analogous to Henry's law for the vapor pressure of a dilute solution.

Leva (5) found that the concentration of the fines had no effect on the

* Tabular material has been deposited as document No. 6259 with the American Documentation. Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or for 35-mm microfilm.

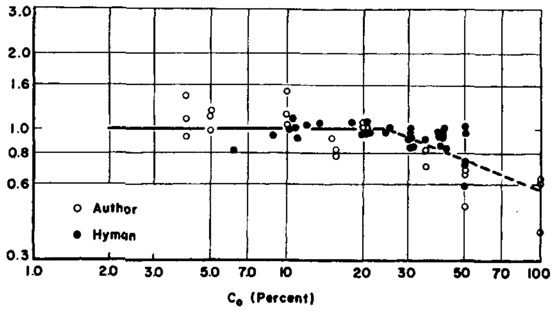

Fig. 8. Effect of fines concentration on the specific elutriation constant.

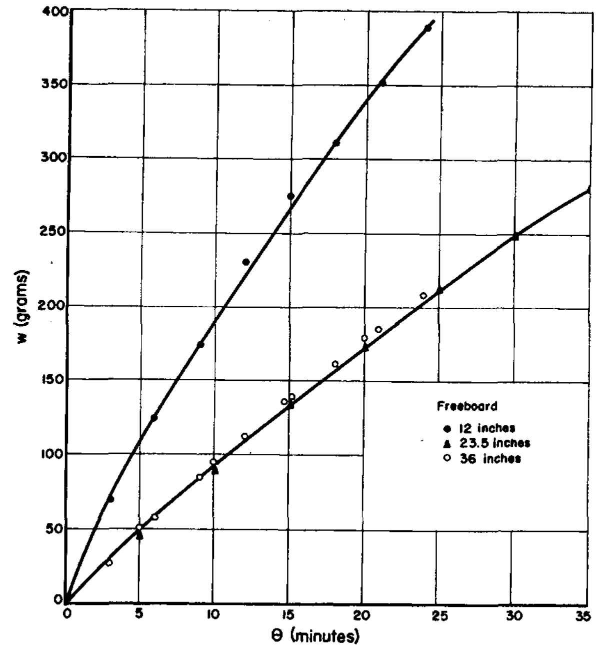

Fig. 9. Effect of freeboard on weight of fines collected.

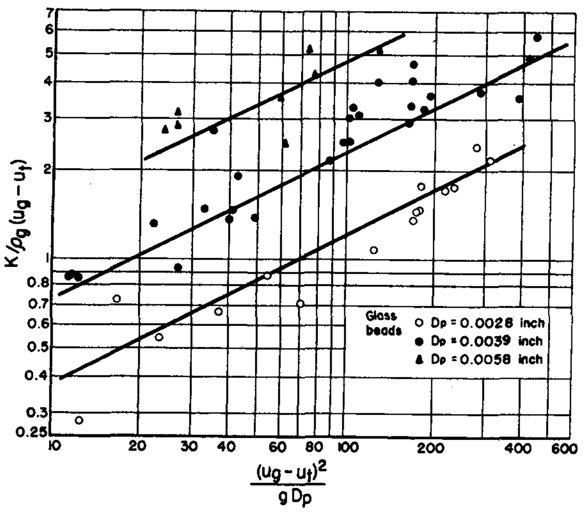

Fig. 10. Effect of Froude number on specific elutriation constant.

value of the elutriation constant from 20 down to 5%, whereas Osberg and Charlesworth (7) noted a significant effect from 5 down to a few tenths %. No significant change on elutriation constant was observed as low as 4% of fines concentration in this investigation.

Based on Figure 8 a correction can be made to estimate roughly the value of K when fines concentration is above 25% with C_o expressed in percentages:

$$K_{>25\%} = K_{<25\%} \left(\frac{C_o}{25} \right)^{-0.48} \quad (9)$$

Freeboard

Freeboard is the distance between the expanded bed and the bed outlet. In Figure 9 the influence of freeboard on the weight of fines removed from a bed is shown. Three different heights, 12, 23.5, and 36 in., were used. The 12-in. height shows a definite increase in the amount of fines collected compared with the 23.5- and 36-in. heights, while equal amounts of fines were collected at the 23.5- and 36-in. heights. It can be seen that there is a critical height above the expanded bed, where the weight of fines collected remains constant. Similar findings were reported by Jolley and Stantan (3). The critical height can be estimated if the length of the column above the bed required to fully develop the gas flow profile is found. No attempt has been made to relate the location of this height to the other system variables.

Coarse-Particle Size

The effect of the coarse-particle size on the value of the elutriation constant was found to be insignificant. Leva (5) concluded that coarse-particle size had a negligible effect on the elutriation constant. Osberg and Charlesworth (7) found that only below a fines concentration of 1% did the coarse-particle size have an effect on the elutriation constant.

Generalized Correlation

Figures 10, 11, and 12 are those used for the evaluation of the constants in the generalized equation. The results of the correlation indicated that the power of the Froude number, $(u_g - u_t)^2/gD_p$, was 0.5; the Reynolds number, $D_p u_t \rho_g/\mu$, was 0.725; and the $(\rho_s - \rho_g)/\rho_g$ group was 1.15. Figure 12 is the final generalized plot including all the data taken in this investigation and the data of Leva (5), Hyman (2), and Yagi and Aoji (12). For fines concentration below 25% the final equation obtained is

$$\frac{K}{\rho_g(u_g - u_t)} = 1.52 \times 10^{-5} \left[\frac{(u_g - u_t)^2}{gD_p} \right]^{0.5} \left[\frac{D_p u_t \rho_g}{\mu} \right]^{0.725} \left[\frac{\rho_s - \rho_g}{\rho_g} \right]^{1.15} \quad (10)$$

where the limits of operation are

0.001595 in. $< D_p <$ 0.0058 in.
0.0103 lb./cu. ft. $< \rho_g <$ 0.075 lb./cu. ft.
81 lb./cu. ft. $< \rho_s <$ 312 lb./cu. ft.
0.725 ft./sec. $< u_g <$ 4.33 ft./sec.

The data of Osberg and Charlesworth (7) were not included in this correlation because of the extremely small particle diameter (approximately 0.00102 in.) and the small fines concentration (below 1%) used. When a fluidized bed is operated at a low velocity, as was the case for Osberg and Charlesworth, there is a tendency for the gas to channel through the bed. The considerable effect of channeling and slugging on the rate of elutriation is evident from a comparison of the value of k operating a fluidized bed with and without a baffle. During the collection of the data for the 2-in. column. slugging was noticed, and a baffle was placed in the column to reduce the influence of slugging on the rate of elutriation. Screens with openings of 5 mesh and spaced approximately 5 in. apart were used as the baffle. The values of the elutriation constant obtained for a 1,500-g and a 1,000-g bed operated with and without the baffle are compared in Figure 13. In the 1,000-g. bed the use of the baffle had no effect on the value of k; in the 1,500-g. bed the baffle increased the value of k. During the operation of the 1,000-g. bed there was insufficient slugging for the baffle to have any effect on the value of the elutriation constant.

The fines concentration used by Osberg and Charlesworth was of such magnitude that if an error were involved in the evaluation of the initial concentration, a much larger error in the value of the elutriation constant would result. If a larger fines concentration is utilized, an error of similar magnitude will have less effect on the value of k.

Lang (4) and Richards (10) reported elutriation of single-particle size system. The data reported by Lang (4) for single-particle size system are compared with two-particle size system of the present investigation in Figure 14. In spite of good agreement, the mechanism of the elutriation for a single-particle size system should be regarded as entirely different from that of multiparticle size system. In a single-particle size system, in order for the particles to be elutriated from the bed,

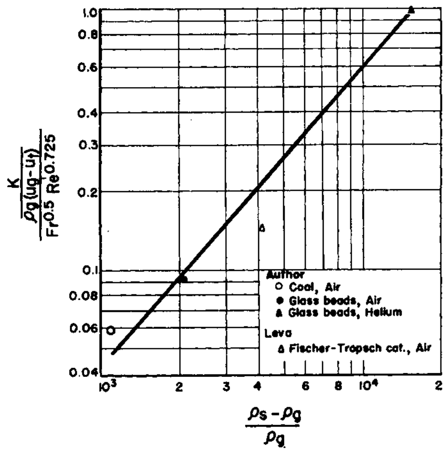

Fig. 11. Effect of solid and gas density on specific elutriation constant.

the gas velocity must be larger than the terminal falling velocity of the particle, and therefore a considerably large expansion of the bed will result. It has been known that the particle movement and the flow patterns for an excessively expanded bed, known as *dilute-phase fluidization*, are quite different from those of dense-phase fluidization. Since the fines concentration is relatively small for the multiparticle size system in the present investigation, the status of fluidization is essentially that of dense phase.

A potential source of experimental error which is frequently encountered in elutriation is an accumulation of an electric charge on the particles in the bed. When the particles become charged, they adhere to the equipment and reduce the accuracy of the work. The smaller the particle, the greater the charge it will accumulate; since the particles employed by Osberg and Charlesworth were small, a static charge on them might have influenced the results. The column was grounded by wrapping the lead wires around the column and also introducing a ground wire into the moving fluidized particles. When the electrostatic charge was sufficiently eliminated from the column, reproducible data were obtained. A good electric conductor may be used as the fluidizing solids to reduce this effect further.

Column Diameter

The effect of column diameter was studied, and the results are shown in Figure 15, where

$$\frac{\dfrac{K}{\rho_g(u_g-u_t)}\left(\dfrac{\rho_g}{\rho_s-\rho_g}\right)^{1.15}}{\left[\dfrac{(u_g-u_t)^2}{gD_p}\right]^{0.5}\left(\dfrac{D_p u_t \rho_g}{\mu}\right)^{0.725}}$$

is plotted against the column diameter. Each point on the plot represents an average of the above group for the corresponding column diameters. The work of Leva (5) in a 1.32-in.–diameter column, of this investigation in a 2- and 4-in. column, and of Hyman (2) in a 3-in. column are included on this plot. The dotted line represents the approximate trend of the column-diameter effect observed by Lang (*4*). More data on larger diameter columns are necessary to make conclusive statements for the effect of column diameter. It is believed that column diameter will have only small effect on the value of the elutriation constant when the diameter of the column is large and whenever channeling and slugging can be eliminated.

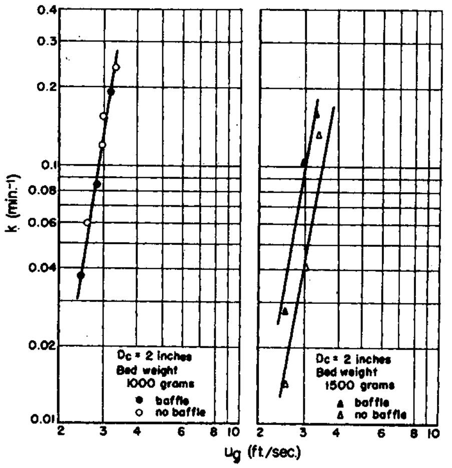

Fig. 13. Effect of a baffle on the elutriation constant.

APPLICATION

To use the new correlation one must be able to evaluate the value of K from the operating conditions and the bed variables. With the value of K known, an equation can be derived to calculate the amount of each size of fine particles elutriated for a specified length of time:

$$C = \frac{w_o - w}{W_t - w}$$

$$\frac{dC}{d\theta} = \frac{w_o - W_t}{(W_t - w)^2}\frac{dw}{d\theta} = -k\frac{w_o - w}{W_t - w}$$

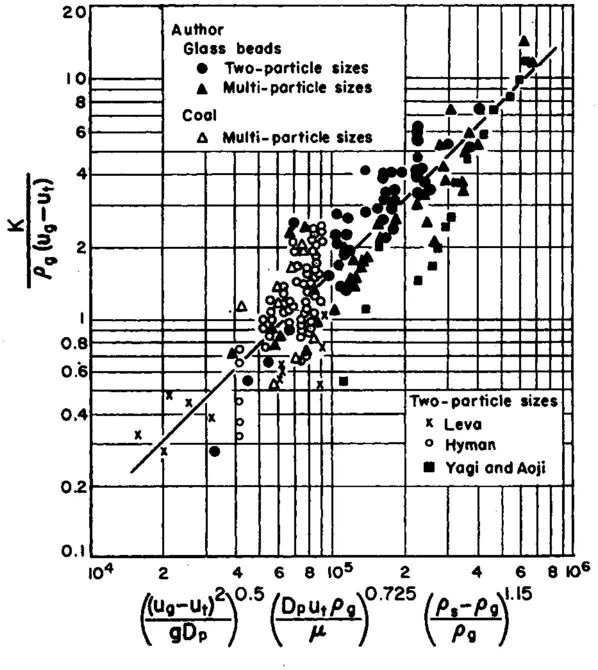

Fig. 12. Generalized plot of proposed correlation.

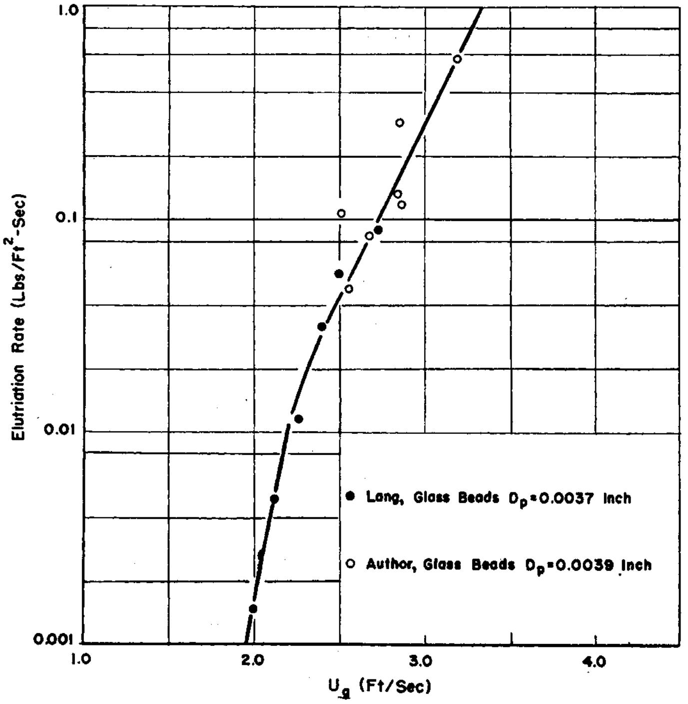

Fig. 14. Comparison of rate of elutriation for single particle size system.

Since $k = \mathrm{K}\, A/W_t - w$

$$\frac{w_o - W_t}{(W_t - w)^2}\frac{dw}{d\theta} = -\frac{KA(w_o - w)}{(W_t - w)^2}$$

$$(w_o - W_t)\int_0^w \frac{dw}{(w_o - w)} = -KA\int_0^\theta d\theta$$

$$w = w_o\,[1 - \exp KA\theta/(w_o - W_t)] \quad (11)$$

For a multiparticle size system W_t is the weight of coarse particles and the weight of those fines in question.

The following example illustrates the application of the developed correlation.

Example

A four-particle size system of glass beads is to be elutriated for 2 min. with air at 70°F. and 1 atm. pressure. Compute the grams of each fine-particle size which will be carried out from the column.

D_p = 0.0028, 0.0039, 0.0058 in.
Weight of the coarse particles (D_p = 0.0110 in.) = 4,050 g.
Concentration of each fine size = 10% or 450 g.
u_g = 4.02 ft./sec.

D column = 4 in.

ρ_s = 156 lb./cu. ft.

Solution

The terminal falling velocities of the fines are calculated by the use of Equations (6), (7), and (8). The values of K are evaluated from Equation (10). From Equation (11) the weight of fines elutriated can be calculated for each particle size:

D_p, in.	u_t, ft./sec.	K, lb./(sq. ft.)(sec.)	Weight actually removed, g.	Weight calculated, g.
0.0028	1.07	1.061	414	320
0.0039	1.88	0.933	291	298
0.0058	2.92	0.354	148	153
			853	771

CONCLUSIONS

A generalized correlation for evaluation of the specific elutriation-rate constant in a two- and multiparticle size system is proposed. In a multiparticle size elutriation the rate of entrainment of each individual-size of fines is independent of the presence of other size particles if that fine size is considered with the coarse size as two-particle size systems. The final dimensionless equation obtained for fines concentration less than 25% is given by Equation (10).

For the fines concentration above approximately 25% a correction should be made.

The variables affecting the elutriation constant are studied, and it is concluded that

1. The difference between the magnitude of the linear gas velocity and particle terminal velocity is the major factor influencing the value of the elutriation constant.
2. For large-diameter columns the bed dimensions and the coarse-particle size have a negligible effect on the value of the specific elutriation constant.
3. In a two-particle size system the initial fines concentration seems to have no effect on the elutriation constant until a concentration of approximately 25% is reached; above 25% the constant tends to decrease as the concentration increases.
4. Freeboard has an influence on the rate of elutriation only when the bed outlet is located below a certain height above the expanded bed.
5. The elutriation constant varies with the gas velocity approximately to the 7.0 power and inversely with the bed weight. The fine particle size, the gas density, and the gas viscosity also influence the value of k.

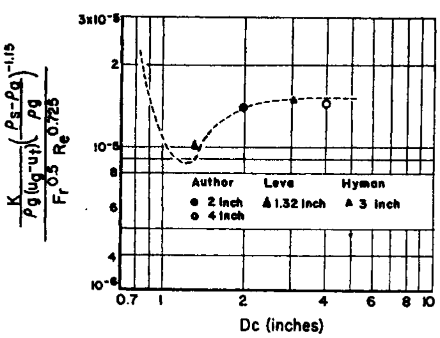

Fig. 15. Effect of column diameter.

NOTATION

a, b, c, d = constants to be evaluated from experimental results, Equation (5)
A = area of column (L^2)
C = concentration of the bed at any time
C_o = initial concentration of the bed
D_p = diameter of particle (L)
f = function of
g = gravitational constant (L/θ^2)
L = bed height (L)
k = elutriation-rate constant ($1/\theta$)
K = specific elutriation-rate constant ($M/\theta L^2$)
K_{avg} = average value of $K(M/\theta L^2)$
u_g = linear gas velocity (L/θ)
u_o = initial velocity of the particle at the surface of the bed (L/θ)
u_t = particle terminal velocity (L/θ)
W = weight of the bed (M)
w_o = initial weight of the fines in the bed (M)
w = weight of the fines elutriated from the bed (M)
W_t = total bed weight considered as two-particle system (M)

Greek Letters

β = constant to be evaluated from experimental results, Equation (5).
ρ_g = gas density (M/L^3)
ρ_s = solid absolute density (M/L^3)
θ = time (θ)
ϕ = particle shape factor defined as area of a sphere divided by area of particle shape having same volume as sphere
μ = gas viscosity ($M/L\theta$)
$\dfrac{(u_g - u_t)^2}{gD_p}$ = Froude number (Fr)
$\dfrac{D_p u_t \rho_g}{\mu}$ = Reynolds number (Re)

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